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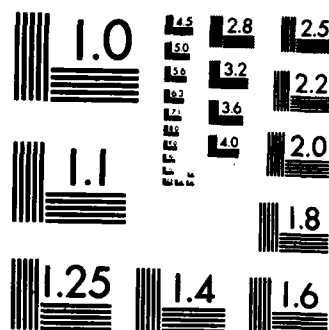
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DEVELOPMENT AND APPLICATION OF THE P-VERSION OF
THE FINITE ELEMENT METHOD, AFOSR-82-0315

INTERIM REPORT, 30 SEP 82-29 ~~SEP~~ 83

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1. DISCUSSION

Two approaches to finite element analysis are now widely recognized in the engineering and mathematical communities. In both approaches the domain Ω is divided into simple convex subdomains (usually triangles or rectangles in two dimensions, and tetrahedra or bricks in three dimensions) and over each subdomain the unknown (displacement field) is approximated by a (local) basis function (usually a polynomial of degree p). Basis functions are required to join continuously at boundaries of the subdomains in the case of planar or 3 dimensional elasticity, or smoothly in the case of plate bending. The difference between the two approaches lies in the manner in which convergence is achieved. These two approaches are:

1. The h-version of the finite element method. In this approach the degree p of the approximating polynomial is kept fixed, usually at some low number such as 2 or 3. Convergence is achieved by allowing h , the maximum diameter of the convex subdomains, to go to zero. Estimates for the error in energy have long been known. In all of these estimates p is assumed to be fixed and the error estimate is asymptotic in h , as h goes to zero.
2. The p-version of the finite element method. In this approach the subdivision of the domain Ω is kept fixed but p is allowed to increase until a desired accuracy is attained. The p-version is reminiscent of the Ritz method for solving partial differential equations but with a crucial distinction between the two methods. In the Ritz method a single polynomial approximation is used over the entire domain Ω (Ω , in general, is not convex). In the p-version of the finite element method polynomials are used as approximations over convex subdomains. This critical difference gives the p-version a much more rapid rate of convergence than either the Ritz method or the h-version.

The p-version of the finite element method requires families of polynomials of arbitrary degree p defined over different geometric shapes. Polynomials defined over neighboring elements join either continuously (are in C^0) for planar or three dimensional elasticity, and smoothly (are in C^1) for plate bending. In order to implement the p-version efficiently on the computer, these families should have the property that computations performed for an approximation of degree p are re-usable for computations performed for the next approximation of degree $p + 1$. We call families possessing this property hierarchic families of finite elements.

The h-version of the finite element method has been the subject of intensive study since the early 1950's and perhaps even earlier. Study of the p-version of the finite element method, on the other hand, began at Washington University in St. Louis in the early 1970's. Research in the p-version (formerly called The Constraint Method) has been supported in part of the Air Force Office of Scientific Research.

Recent Research Accomplishments are:

1. The proof a theorem which establishes that for plate bending problems (in which C^1 -continuity is required) the rate of convergence of the p-version is twice that of the h-version. This supplements an earlier theorem which was proved for the C^0 case.
2. Hierarchic families of C^1 -triangular elements have been implemented on the computer using pre-computed arrays.
3. The difficult problem of the finite element analysis of a rhombic plate has been solved using both the p-version and the h-version.
4. Work has begun on an efficient post-processing technique to compute accurate displacements and stresses, and to calculate stress intensity factors.

3. PROFESSIONAL PERSONNEL

1. I. Norman Katz, Professor of Applied Mathematics and Systems Science, Washington University
2. Barna A. Szabo, A. P. Greensfelder Professor Civil Engineering, Washington University
3. Xing-Ren Ying, Research Assistant, doctoral candidate in the Department of Systems Science and Mathematics, Washington University
4. Hong-Cai Wang, Research Assistant, doctoral candidate in the Department of Systems Science and Mathematics, Washington University

3. PAPERS PUBLISHED AND PRESENTED SINCE THE START OF THE PROJECT (1977)

3.1 Published Papers:

1. "Hierarchal Finite Elements and Precomputed Arrays", by Mark P. Rossow and I. Norman Katz, Int. J. for Num. Method in Engr., Vol. 13, No. 6 (1978) pp. 977-999.
2. "Nodal Variables for Complete Conforming Finite Elements of Arbitrary Polynomial Order", by I. Norman Katz, A. G. Peano, and Mark P. Rossow, Computers and Mathematics with Applications, Vol. 4, No. 2, (1978), pp. 85-112.
3. "Hierarchic Solid Elements for the p-version of the Finite Element Method", by I. Norman Katz, B. A. Szabo and A. G. Peano (in preparation).
4. "P-convergence Finite Element Approximations in Linear Elastic Fracture Mechanics", by Anil K. Mehta (doctoral dissertation), Department of civil Engineering, Washington University (1978).
5. "An Improved p-version Finite Element Algorithm and a Convergence Result for the p-version" by Anthony G. Kassos, Jr. (doctoral dissertation) Department of Systems Science and Mathematics, Washington University, (August, 1979).
6. "Hierarchic Families for the p-version of the Finite Element Method", I. Babuska, I. N. Katz and B. A. Szabo, invited paper presented at the Third IMACS International Symposium on Computer Methods for Partial Differential Equations, published in Advances in Computer Methods for Partial Differential Equations - III (1979) pp. 278-286.
7. "The p-version of the Finite Element Method", I. Babuska, B. A. Szabo, and I. N. Katz, SIAM J. of Numerical Analysis Vol. 18, No. 3, June 1981 pp. 515-545.
8. "Hierarchic Triangular Elements with one Curved Side for the p-version of the Finite Element Method", by I. Norman Katz (in preparation).
9. "The p-version of the Finite Element Method for Problems Requiring C^1 -Continuity", by Douglas W. Wang (doctoral dissertation), Department of Systems Science and Mathematics, Washington University August 1982.
10. "Implementation of a C^1 Triangular Element based on the p-version of the Finite Element Method", by I. Norman Katz, D. W. Wang and B. Szabo, Proceedings of the Symposium in Advances and Trends in Structural and Solid Mechanics, October 4-7, 1982, Washington, D.C.
11. "The p-version of the Finite Element Method for Problems Requiring C^1 -Continuity", by I. Norman Katz and Douglas W. Wang (submitted for publication).
12. "Implementation of a C^1 -triangular Element based on the p-version of the Finite Element Method" by Douglas W. Wang, I. Norman Katz and Barna A. Szabo, accepted for publication in Computers and Structures.
13. "h- and p- version analyses of a Rhombic Plate" by Douglas W. Wang, I. Norman Katz and Barna A. Szabo, accepted for publication in Int. J. for Num. Methods in Eng.

3.2 Presented Papers:

1. "Hierarchical Approximation in Finite Element Analysis", by I. Norman Katz, International Symposim on Innovative Numerical Analysis in Applied Engineering Science, Versailles, France, May 23-27, 1977.
2. "Efficient Generation of Hierarchal Finite Elements Through the Use of Precomputed Arrays", by M. P. Rossow and I. N. Katz, Second Annual ASCE Engineering Mechanics Division Speciality Conference, North Carolina State University, Raleigh, NC, May 23-25, 1977.
3. " C^1 Triangular Elements of Arbitrary Polynomial Order Containing Corrective Rational Functions", by I. Norman Katz, SIAM 1977 National Meeting, Philadelphia, PA, June 13-15, 1977.
4. "Hierarchical Complete Conforming Tetrahedral Elements of Arbitrary Polynomial Order", by I. Norman Katz, presented at SIAM 1977 Fall Meeting, Albuquerque, NM, October 31- November 2, 1977.
5. "A Hierarchical Family of Complete Conforming Prismatic Finite Elements of Arbitrary Polynomial Order", by I. Norman Katz, presented at SIAM 1978 National Meeting, Madison, WI, May 24-26, 1978.
6. "Comparative Rates of h- and p- Convergence in the Finite Element Analysis of a Model Bar Problem", by I. Norman Katz, presented at the SIAM 1978 Fall Meeting, Knoxville, Tennessee, October 20- November 1, 1978.
7. "Smooth Approximation to a Function in $H_0^2(D)$ by Modified Bernstein Polynomials over Trangles" by A. G. Kassos, Jr. and I. N. Katz, presented at the SIAM 1979 Fall Meeting, Denver, Colorado, November 12-14, 1979.
8. "Triangles with one Curved Side for the p-version of the Finite Element Method" by I. Norman Katz, presented at the SIAM 1980 Spring Meeting, Alexandria, VA, June 5-7, 1980.
9. "Hierarchic Square Pyramidal Elements for the p-version of the Finite Element Method" by I. Norman Katz, presented at the SIAM 1980 Fall Meeting, Houston, TX, November 6-8, 1980.
10. "The Rate of Convergence of the p-version of the Finite Element Method for Plate Bending Problems", by Douglas W. Wang and I. Norman Katz, presented at SIAM 1981 Fall Meeting, October 6-8, 1981, Cincinnati, Ohio.
11. "The p-version of the Finite Element Method", by I. Norman Katz, 1982 Meeting of the Illinois Section of the Mathematical Association of America, Southern Illinois University at Edwardsville, April 30-May 1, 1982.
12. "Computer Implementation of a C^1 Triangular Element based on the p-version of the Finite Element Method", by Douglas W. Wang and I. Norman Katz, SIAM 30th Anniversary Meeting, July 19-23, 1982, Stanford, California.

13. "Implementation of a C^1 Triangular Element Based on the p-version of the Finite Element Method", Symposium on Advanced and Trends in Structural and Solid Mechanics, October 4-7, 1982, Washington, D.C.
14. "P-Convergent Polynomial Approximations in $H_0^2(\Omega)$ " by Douglas W. Wang and I. Norman Katz, Fourth Texas Symposium on Approximation Theory. Department of Mathematics, Texas A & M University, College Station Texas 77843, January 17-21, 1983.
15. "Design Aspects of Adaptive Finite Element Codes" by D. W. Wang, I. N. Katz and M. Z. Qian, to be presented at the ASCE-EMD (American Society of Civil Engineers-Engineering Mechanics Division) Speciality Conference, Purdue University, May 28-25, 1983.
16. "Smoothing Stresses Computed Pointwise by the p-version of the Finite Element Method" by I. Norman Katz and Xing-ren Ying, presented at SIAM 1983 National Meeting, Denver, Colorado, June 6-8, 1983.

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